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ROTATIONAL DECAY OF THE SATELLITE 1960 ETA 2 DUE TO THE EARTH'S MAGNETIC FIELD

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SUMMARY

Observations of the signal null rate of the Solar Radiation I satellite (1960 η2) launched with Transit II-A (1960 η1), show a rotational decay, primarily exponential, with a relaxation time of 66 days. It is of interest to ascertain that, with numerical changes in the formulas assumed for the rotational decay of earlier satellites, the rotational decay of this satellite also (as that of earlier satellites) implies a mean geomagnetic field which agrees with that inferred from ground measurements. A new factor which enters here is that linear damping by magnetic hysteresis was more prominent than the exponential damping by eddy currents during the last few weeks before April 8, 1961, when the satellite is calculated to have stopped rotating.

CONTENTS

List of Symbols

a — semimajor axis of an elliptical orbit

B — the induced magnetic flux in a substance

C — mechanical rotational couple or torque

C_h — couple which is due to magnetic hysteresis

e — eccentricity of a satellite's orbit; also base of natural logarithms = 2.718...

H — magnetic field strength. Since the present discussion is concerned with geomagnetism, its unit is the geophysical gauss

$\overline{H}_{\perp}$ — mean magnetic field component normal to the satellite's spin axis

$\overline{H}_H$ — mean, under a satellite orbit, of the horizontal magnetic field at Earth's surface

$\overline{H}_V$ — mean, under a satellite orbit, of the vertical magnetic field at Earth's surface

$\overline{H}_o$ — mean, under a satellite orbit, of the total magnetic field at Earth's surface

$\overline{H}$ — mean, in a satellite orbit, of the total geomagnetic field

h — height or length of a right cylinder

I — rotational moment of inertia of a body

i — inclination of the satellite's orbital plane to Earth's equator

k — ratio of diameter to length of a right circular cylinder

M — mean anomaly of a satellite in its orbit. It is an angle measured from perigee and is the arc of a complete circle corresponding to the proportion of orbital period elapsed since the previous perigee passage

r — radius of any body of revolution

Δr — thickness of a shell

t — an instant of time

t_0 — launch date of Solar Radiation I

V — total volume of ferromagnetic material involved in magnetic hysteresis effects

α — angle between the vertical and the satellite spin axis

θ — angle between the mean direction of the satellite spin axis, assumed to be horizontal at the injection point of the orbit, and the horizontal at any point of the orbit; also, the angle between the external magnetic field and a dipole line

κ — magnetic susceptibility of satellite material involved in hysteresis effects

μ — effective magnetic permeability

σ — electrical conductivity, the reciprocal of resistivity

ϕ — spherical coordinate of latitude

ω — angular velocity in rotations per second

Laboratory (NRL) and launched with Transit II-A (1960 η1) (Figures 1 and 2). A semilogarithmic plot of the observed rotation rate vs. time, for the first four months in orbit, shows an exponential decay represented by

$$\omega = 0.77 \exp\left(\frac{t_0 - t}{5.7 \times 10^6}\right) \frac{\text{rotations}}{\text{sec}} , \tag{1}$$

where t_0 = June 22, 1960 is the launch date, and $t_0 - t$ is in seconds (Figure 3). Thus the relaxation time (for spin-rate division by $e = 2.718...$) is about 66 days, which is nearly equal to the relaxation time of 72 days found for Vanguard II, a satellite for which structural details were quite similar. Deviations of data points from the exponential line in the left half of Figure 3 may be partly explained by variations of electrical conductivity of the satellite consequent to variations of the percentage of time in sunlight (Figure 4).

The damping coefficient for this exponential decay follows from the above observations and from mechanical principles:

$$\frac{C}{\omega} = \frac{I}{\text{relaxation time}} = \frac{5.396 \times 10^6}{5.702 \times 10^6} = 0.9465 \frac{\text{gm-cm}^2}{\text{sec}} , \tag{2}$$

where C is the mechanical rotational couple, and I is the moment of inertia about the spin

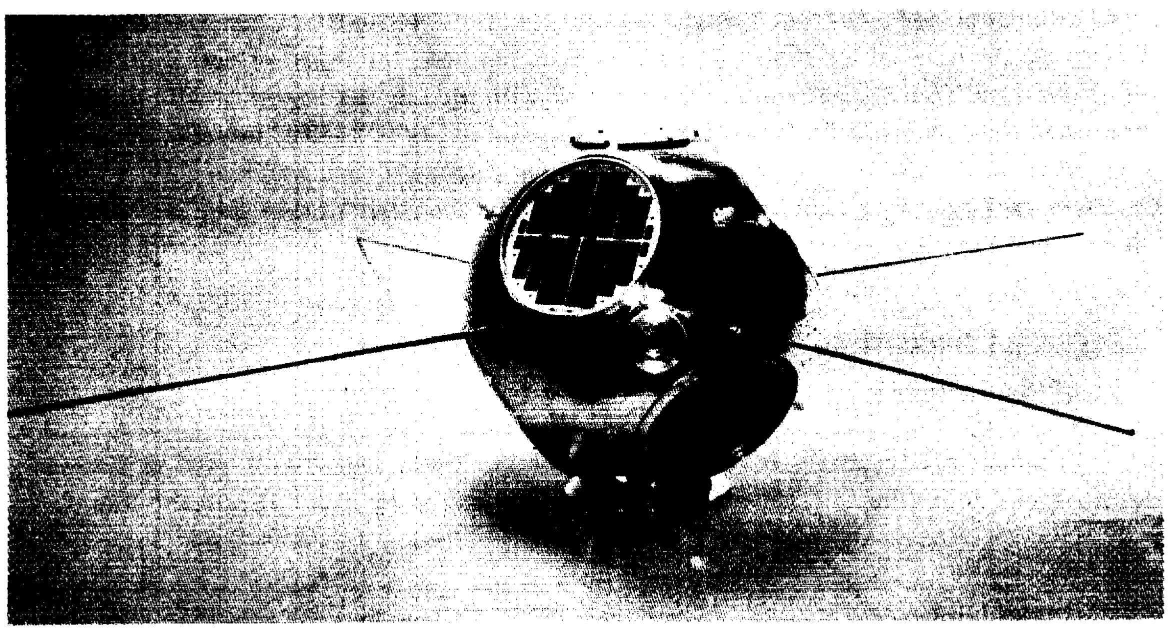

Figure 1—Prototype of Solar Radiation I Satellite. The magnet and the X-ray sensor mount can be seen.

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INTRODUCTION

Satellites of approximately spherical shape can be expected to show the clearest evidence of magnetic torques, because this shape is least susceptible to disturbance by other torques, such as those due to aerodynamic and gravitational forces. Thus, the observed rotational decay of the satellites Vanguard I (1958 β 2) and Vanguard II (1959 α1) have been shown to be explained, with fair quantitative precision, by the theory of induced eddy-current torques acting on the electrically conducting parts of each satellite (References 1 and 2). Now a further confirmation of this magnetic damping theory comes from the satellite Solar Radiation I (1960 η2), whose declining rotation rate was accurately observed almost daily from the time of its launching on June 22, 1960, until late in February 1961, when the rate became too slow for practical observation. The present study shows that the theoretical eddy-current torque on various conducting parts of this satellite does indeed explain most of its observed rotational decay, and the mean effective geomagnetic field deduced from the calculation agrees well with the standard geomagnetic dipole model following the inverse cube law. Also, the rotational decay of 1960 η2 has a feature of special interest: evidence of a small additional linear decay torque which seems to be quantitatively explained by magnetic hysteresis effects on the parts of this satellite known to be ferromagnetic.

OBSERVED EXPONENTIAL DECAY AND CORRESPONDING DAMPING TORQUES

Frequent observations have been made of the radio signal null rate of the satellite Solar Radiation I (1960 η2), a sphere constructed by the U. S. Naval Research

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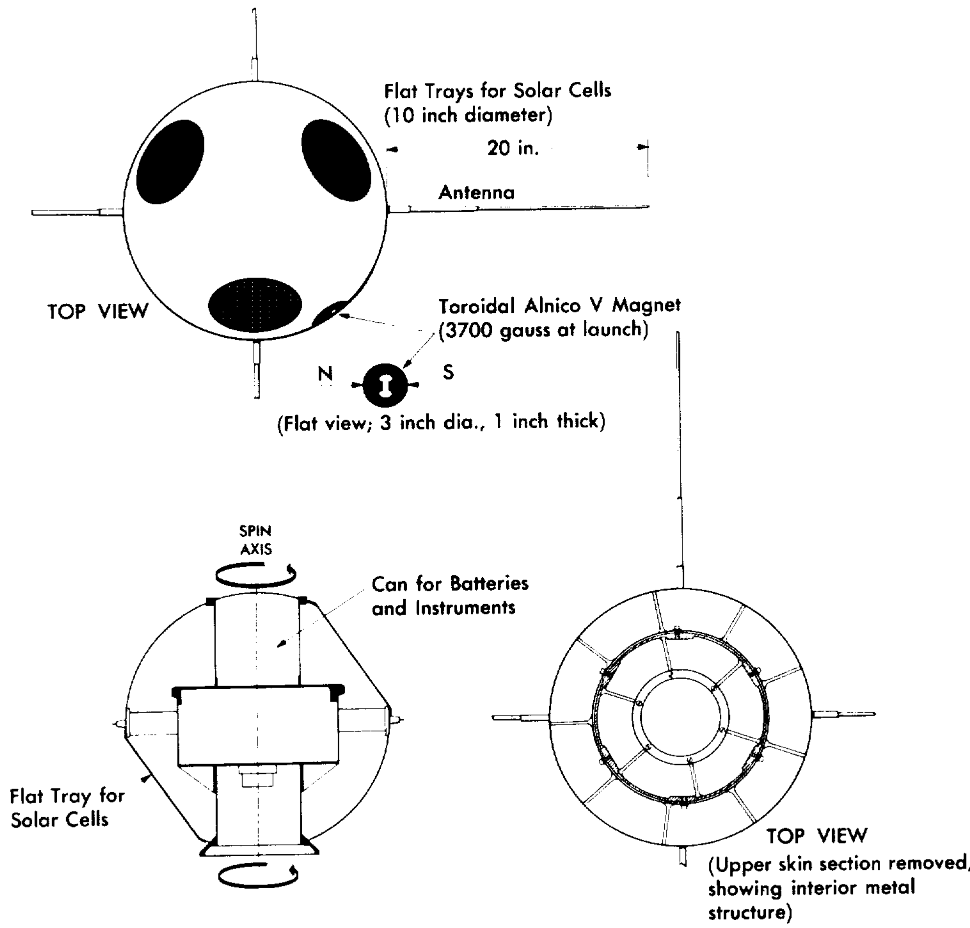

Figure 2—Schematic diagram of Solar Radiation I (cf. Figure 1)

axis measured before launching. This value of the braking couple depends entirely on observed quantities, and is thus independent of any theory concerning its cause.

THEORETICAL EDDY-CURRENT DAMPING TORQUES AND THE SOLUTION FOR THE GEOMAGNETIC FIELD

From the theory of eddy-current induction and the consequent magnetic damping, the expected numerical value of $C/\omega/\overline{H}_{\perp}^{2}$ for the conducting parts of the satellite may be solved (Reference 1). For the similar satellite Vanguard II, this quantity was computed as 6.364 gm-cm^2/sec-gauss^2 for the outer shell. However, the outer shell of Solar Radiation I differs from that of Vanguard II: (1) it is made of commercial aluminum having a resistivity of $1/\sigma$ = 5500 emu (a resistivity of 10,000 emu has been assumed for the magnesium shell of Vanguard II); (2) it had six zones-of-one-base, having diameters equal to the radius of

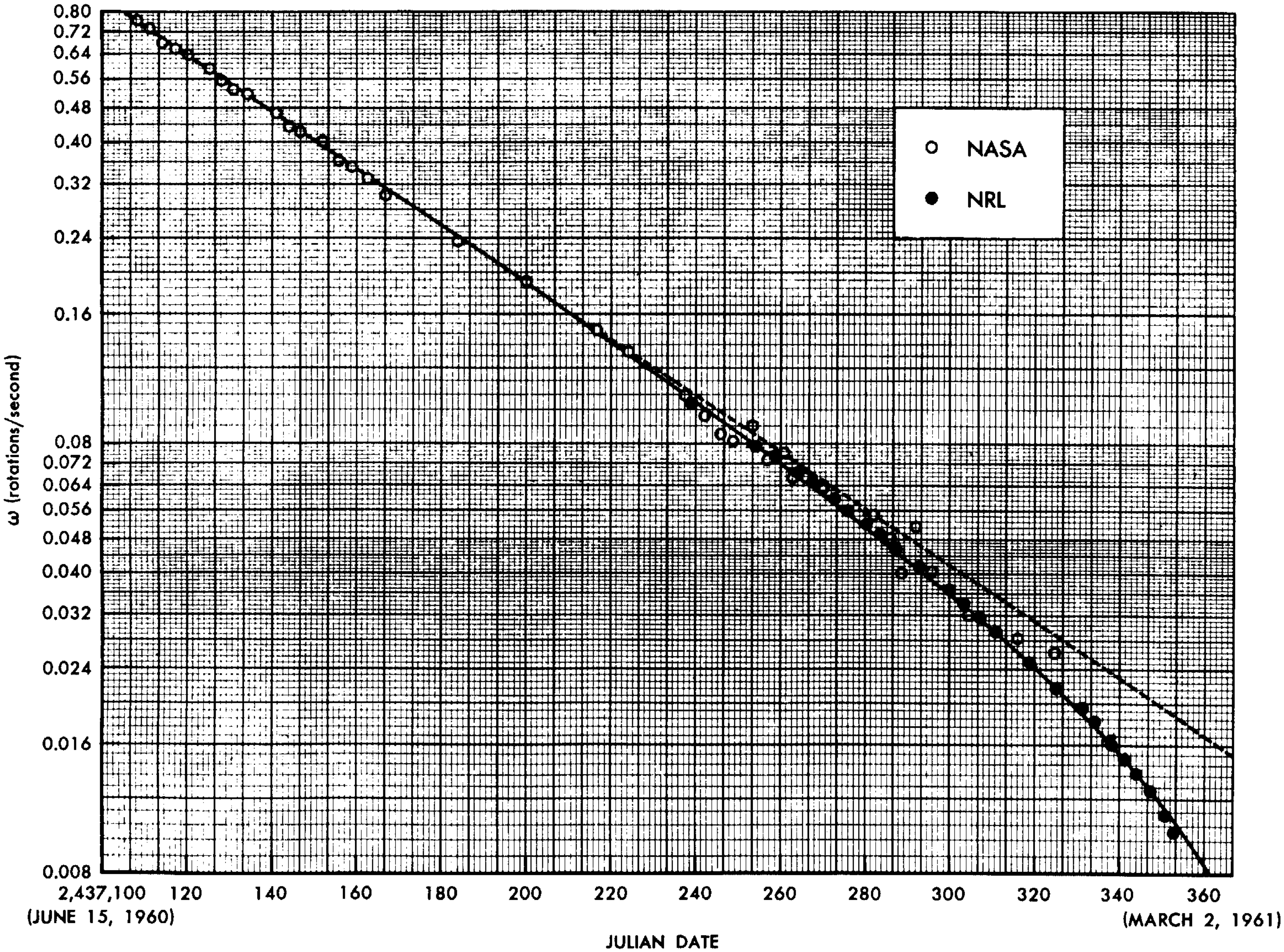

Figure 3—Observed spin rate vs. time for Solar Radiation I. The straight line represents exponential decay with a relaxation time of 66 days. The curve includes additional linear decay of 3.3×10^{-5} rotations per second per day. Extrapolation shows rotation to have stopped on J.D. 2,437,397 (April 8, 1961)

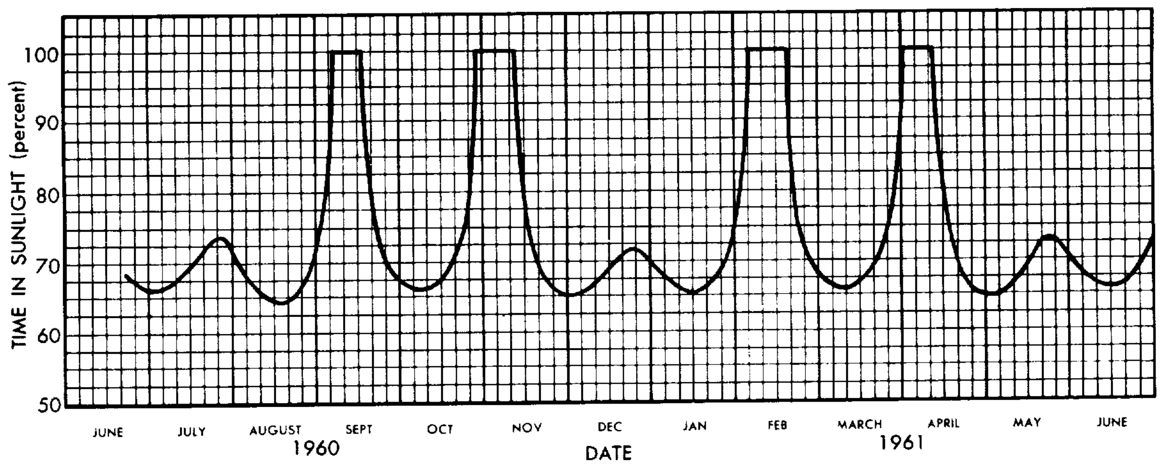

Figure 4—Percentage of time Solar Radiation I spent in sunlight for approximately one year (predicted from its orbit)

the satellite, cut from its spherical conducting area, so the spherical conducting area is $3\sqrt{3}/2 - 2$ or 0.6 that of Vanguard II; (3) its thickness is 0.1 cm (0.073 cm for Vanguard II). Using these values as corrections to the magnetic damping coefficient of Vanguard II, we find

$$\frac{C_1}{\omega} = 6.364 \frac{0.6 \times 10 \times 10}{5.5 \times 7.3} \overline{H}_1^2$$

$$= 9.510 \, \overline{H}_1^2 \, \frac{\text{gm-cm}^2}{\text{sec-gauss}^2} , \qquad (3)$$

for the outer shell of 1960 η2. The six zones-of-one-base cut from the continuous outer shell were replaced by flat circular aluminum trays supporting, on insulation, the solar batteries on the outside of the satellite (Figure 1). Since these six aluminum trays are not insulated from the spherical outer shell, the additional conducting volume proportional to their thickness (0.075 cm) and to their area (3/8 the total area of the sphere) should be similarly computed and included in the total couple of the outer shell:

$$\frac{C_2}{\omega} = 6.364 \frac{0.375 \times 10 \times 7.5}{5.5 \times 7.3} \overline{H}_1^2$$

$$= 4.458 \, \overline{H}_1^2 \, . \qquad (4)$$

The internal instrument package (Figure 2), also made of commercial aluminum for which $1/\sigma = 5500$ emu, may be analyzed as:

Part A – two closed cylindrical shells of radius $r = 8.4$ cm and length $h = 2r = 16.8$ cm, with shell thickness $\Delta r = 0.1$ cm;

Part B – a central cylindrical shell for which $h = 16$ cm, $r = 15.75$ cm, and $\Delta r = 0.1$ cm.

It has been shown (Equation 13 of Reference 1) that for cylinders having $h = 2r$ and the spin axis parallel to the geometrical axis, the magnetic damping couple is given by:

$$\frac{C}{\omega} = \left(18\pi - \frac{160}{3}\right)\sigma\mu^2 \overline{H}_{\perp}^2 r^4 \Delta r , \tag{5}$$

which yields

$$\frac{C_3}{\omega} = 0.583\ \overline{H}_1^2 , \tag{6}$$

for the two cylinders of part A where μ is unity. The couple for a cylinder in which $k = 2r/h > 1$ is given by (Equation 12 of Reference 1)

$$\frac{C}{\omega} = \sigma\mu^2\ \overline{H}_1^2\ h^4 k \left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2 - 2k}\right) - \frac{3 - 4k}{2k(1-k)\sqrt{k^2 - 1}} \ln\left(k + \sqrt{k^2 - 1}\right)\right] \Delta r , \tag{7}$$

which, computed for the cylinder of part B (with μ = unity), gives

$$\frac{C_4}{\omega} = 3.096\ \overline{H}_1^2 . \tag{8}$$

Other aluminum parts of this satellite which would suffer significant eddy-current couples are the four external antennas and the internal structural tubing (Figure 2). These may be treated as long cylindrical shells with the spin axis perpendicular to the geometrical axis. The formula for the damping coefficient for these has been derived in Reference 1 (Equations 19 and 11):

$$\frac{C}{\omega} = \frac{\sigma \overline{H}_{\perp}^{\,2}}{4}\left\{\pi\mu_1^2 r^3 \left(\frac{r}{2} + h\right) + 2\mu_2^2 h^4 k \left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2 - 2k}\right) - \frac{3 - 4k}{k(1-k)\sqrt{1 - k^2}} \arctan \frac{\sqrt{1 - k^2}}{1 + k}\right]\right\} \Delta r , \tag{9}$$

where the subscripts 1 and 2 indicate magnetic permeabilities effective on fields respectively normal and parallel to the geometrical axis. Thus, for all the antennas

$$\frac{C_5}{\omega} = 0.070\ \overline{H}_1^2\ , \tag{10}$$

and for all the structural tubing,

$$\frac{C_6}{\omega} = 0.010\ \overline{H}_1^2\ . \tag{11}$$

Since all parts considered so far are made of nonferromagnetic materials, the magnetic permeability μ has been assumed to be unity in the computations.

The electrical power system of this satellite includes nine nickel-cadmium cells, mounted so that their cylindrical axes are parallel to the satellite's spin axis. Each cell is a closed cylinder having dimensions of $h = 5.7$ cm and $r = 1.6$ cm and made of malleable iron enclosing a rolled sheet of Ni-Cd, for all of which a mean resistivity $1/\sigma = 10^4$ emu has been assumed. The apparent magnetic permeability for a cross-axis field on each can has been measured as $\mu = 3$ by the U. S. National Bureau of Standards. For such a rotating cylinder Equation 11 of Reference 1 offers a general equation for the damping coefficient:

$$\frac{C}{\omega} = \sigma\mu^2\ \overline{H}_1^2\ h^4 k \left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2 - 2k}\right) - \frac{3 - 4k}{k(1 - k)\sqrt{1 - k^2}} \arctan \frac{\sqrt{1 - k^2}}{1 + k}\right] \Delta r\ , \tag{12}$$

which may be integrated numerically to give a computed total for all nine cans:

$$\frac{C_7}{\omega} = 3.063\ \overline{H}_1^2\ . \tag{13}$$

Finally, there is an Alnico V permanent magnet mounted on the equator of the satellite (pole faces visible in Figure 1), at the solar radiation intake slot, to exclude charged particles. It may be considered as a solid cylindrical disc with $h = 2.5$ cm, $r = 3.8$ cm, $1/\sigma = 5 \times 10^4$, and $\mu = 5$. The general magnetic damping coefficient may be derived by integrating Equation 17 of Reference 1:

$$\frac{C}{\omega} = \frac{\pi\sigma\mu^2\ \overline{H}_1^2\ hr^4}{16}\ , \tag{14}$$

from which it may be computed that

$$\frac{C_8}{\omega} = 0.050\ \overline{H}_1^2 \tag{15}$$

for the permanent magnet. The remaining ferromagnetic material of this satellite, which consists of several small transformer and relay cores, has a higher permeability than the permanent magnet, and may be estimated to have seven times as great a couple:

$$\frac{C_9}{\omega} = 0.350\ \overline{H}_1^2\ . \tag{16}$$

The total of these theoretical couple coefficients must equal that from the damping actually observed. Therefore we solve the equation

$$\sum_{n=1}^{9} \frac{C_n}{\omega} = 21.190\ \overline{H}_1^2 = 0.9465\ , \tag{17}$$

to find $\overline{H}_\perp = 0.211$ gauss as the mean magnetic field component normal to the satellite's spin axis.

COMPARISON OF THE GEOMAGNETIC FIELD IN ORBIT, BASED ON GROUND SURVEYS, WITH THE THEORETICALLY DERIVED VALUE

It is of interest to compare the value derived for $\overline{H}_\perp$ with the mean effective magnetic field strength expected from ground measurements of the geomagnetic field combined with the known dimensions and orientation of the Solar Radiation I orbit (References 2 and 3). In this discussion it is assumed that the mean orientation of the satellite spin axis has been that of a line tangent to its orbit at the point of injection, with the real axis nutating with an observed amplitude of 75 degrees in a period of 18 days because of geomagnetic restoring torques on the 3000-gauss double-horseshoe toroidal permanent magnet. Since most of the plotted points in Figure 3 represent averages of several observations spread over a week or more, the effect of axis nutation is greatly smoothed in this figure.

If we assume the mean latitude range to be measured by the satellite's orbital inclination with respect to the earth's equator, $i = 66\overset{\circ}{.}769$, the mean horizontal surface magnetic vector is:

$$\begin{aligned} \overline{H}_H &= 0.304 \left(\frac{180}{66.769\,\pi}\right) \int_0^{66\overset{\circ}{.}769} \cos\phi \, d\phi \\ &= 0.240 \text{ gauss}\ , \end{aligned} \tag{18}$$

where ϕ is the spherical coordinate of latitude. The corresponding vertical vector is

$$\overline{H}_V = 0.304 \left(\frac{360}{66.769\,\pi}\right) \int_0^{66.^\circ 769} \sin\phi\, d\phi$$

$$= 0.316 \text{ gauss} . \tag{19}$$

Thus the total mean surface field under the orbit is

$$\overline{H}_o = \sqrt{(0.240)^2 + (0.316)^2} = 0.397 \text{ gauss} . \tag{20}$$

If a is the semimajor axis of the orbit, e the eccentricity, and we assume the October 10, 1960, orbital elements as $a = 1.131$ earth radii and $e = 0.031$, and the general time mean-inverse-cube radius as $1/a^3\,(1 - e^2)^{\frac{3}{2}}$, the mean total magnetic field of the Solar Radiation I orbit is

$$\overline{H} = \frac{0.397}{1.4425644} = 0.275 \text{ gauss} . \tag{21}$$

From Equations 18 and 19, the ratio $\overline{H}_V/\overline{H}_H$ is 1.32, and the mean effective field is related to the spin-axis orientation by

$$\overline{H}_\perp = 0.275 \sqrt{\frac{(1.32)^2\,\overline{\sin^2\alpha} + \overline{\sin^2\theta}}{1 + (1.32)^2}} . \tag{22}$$

The angle between $\overline{H}_V$ and the spin axis is α; for any nearly circular orbit, $\overline{\sin\alpha} = 2/\pi$. Also, θ is the angle between $\overline{H}_H$ and the spin axis at injection; it is given by

$$\overline{\cos\theta} = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos M\, dM}{\sqrt{1 + \cot^2 i\ \sec^2(180^\circ - \omega + M)}} . \tag{23}$$

Here the original argument of perigee, assumed to be the injection point, was such that $(180^\circ - \omega) \approx 50$ degrees, M is the general mean anomaly of the satellite in orbit, and i is the inclination of the orbital plane with respect to the equator. Thus, $\overline{\cos\theta}$ averages 0.357, so that $\overline{\sin^2\theta} = 0.8724$, and by Equation 22, $\overline{H}_\perp = 0.209$ gauss. This value, based on surface measures and orbital data, agrees well with that inferred from observed exponential spin decay and assumed eddy-current damping. Therefore it can certainly be concluded that magnetic damping in an inverse cube geomagnetic field was the chief cause of the observed spin decay of satellite 1960 η2.

OBSERVATIONAL AND THEORETICAL EVIDENCE FOR HYSTERESIS DAMPING

There is a very special feature of the spin-rate curve of this satellite, namely that on a semilogarithmic plot, as in Figure 3, the right half has a downward curvature. This became increasingly conspicuous after December 1, 1960, when the satellite's rotation period had exceeded 10 seconds. Such downward curvature is to be expected if an additional damping couple, which is due to magnetic hysteresis, has become significant, since hysteresis decay would be linear rather than exponential (Reference 1, pages 530 and 534). Indeed, all the spin-rate observations are well represented by adding a linear term to Equation 1; so the complete empirical equation is

$$\omega = 0.77 \exp\left(\frac{t_0 - t}{5.7 \times 10^6}\right) + (3.8 \times 10^{-10})\,(t_0 - t)\,\frac{\text{rotations}}{\text{sec}} . \tag{24}$$

An important consequence of even slight linear damping is that the spin rate would reach zero in a finite time period. Thus, by setting the right side of Equation 24 equal to zero, and solving for t, it is found that Solar Radiation I stopped rotating on April 8, 1961.

Unlike the magnetic damping from eddy currents, which exerts a braking couple on all conducting material, hysteresis damping can affect only ferromagnetic parts of a satellite. Hysteresis of a ferromagnetic mass is the lag of magnetic flux **B**, along any given polar line within the mass and normal to the rotation axis, corresponding with any change of external magnetic field **H** along this same polar line. Hence, if such change of **H** is due to its rotation with respect to any given body polar line, the field at any point of the cycle will exert a torque on the body flux, since the latter, because of hysteresis, would correspond to the field **H** rotated by some angle θ between **H** and the given polar line. Since the component for torque on this polar line is $H \sin\theta$, and, for material with magnetic susceptibility κ, the magnetic moment on **H** of the displaced flux per cm^3 of ferromagnetic material is $\kappa H \cos\theta$, the mean couple on a volume V over a complete cycle is (Reference 4):

$$C_h = \frac{V \kappa H^2 \int_0^{2\pi} \sin\theta \cos\theta \, d\theta}{\int_0^{2\pi} d\theta}$$

$$= \frac{-V \kappa H^2}{8\pi} \text{ dyne-cm} . \tag{25}$$

If we replace the magnetic susceptibility κ by its equivalent $(\mu - 1)/4\pi$, and divide by 980.7 for consistency with a moment of inertia I given in $gm\text{-}cm^2$, the equation of rotational motion is

$$I \frac{d\omega}{dt} = -0.001019 \frac{V(\mu - 1) H^2}{32\pi^2} , \tag{26}$$

where H is magnetic field strength. Integrating Equation 26 we find the general hysteresis damping term

$$0.001019 \frac{V(\mu - 1) H^2 (t_0 - t)}{32\pi^2 I} , \tag{27}$$

for $t_0 - t$ in seconds. If, instead of the apparent magnetic permeability μ, which varies considerably over a cycle, the area of the hysteresis loop $\oint \mathbf{H}\, d\mathbf{B}$ for the given material and a range of $\mathbf{H}$ is more conveniently available, the general hysteresis damping term may be written

$$0.001019\ V(t_0 - t) \oint \frac{\mathbf{H}\, d\mathbf{B}}{8\pi^2}\ I . \tag{28}$$

Since, in the case of Solar Radiation I, $I = 5.396 \times 10^6$ gm-cm^2 and the mean effective field $\overline{H}_\perp = 0.21$ gauss, the hysteresis term may be written:

$$(2.63 \times 10^{-14}) V (\mu - 1)(t_0 - t) , \tag{29}$$

where V is in cm^3. Since the observed coefficient of $t_0 - t$ in Equation 24 is 3.8×10^{-10},

$$V(\mu - 1) = 1.4 \times 10^4 \text{ cm}^3 \tag{30}$$

must describe the total ferromagnetic material in this satellite. Hence, 600 cm^3 or about 4 kg of material, having a mean apparent magnetic permeability $\mu = 24$ could explain the observed linear damping as an effect of magnetic hysteresis. The toroidal permanent magnet alone would account for about a sixth of this volume, and the ferromagnetic battery material has a total volume of about 400 cm^3 with a mean μ of 20. The transformer and relay cores would make a contribution of the same order. Thus, the assumptions on the structure of 1960 η2 for both types of magnetic damping, eddy-current and hysteresis, also have mutual quantitative agreement.

So far as known to the author, this satellite is the first case for which there is clear observational evidence of hysteresis damping, except for the Transit satellites which, as described by Fischell (Reference 5), were purposely designed to have large and dominant hysteresis damping.

CONCLUSION

The fact that the hysteresis factor of magnetic damping is linear with time, so that at slower rotational speeds it exceeds the eddy-current factor, surely indicates that this factor is highly important for any application of magnetic damping to special designs for steering space vehicles. Thus, after the control and slowing of rapid angular rotations and librations by eddy-current effects, the slower angular speeds could be steadily damped down to zero with increasing relative dominance of the hysteresis effect, as exemplified by the increasing downward curvature of spin rates in Figure 3.

ACKNOWLEDGMENTS

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